Title: Watts Up, Doc?

Brief Overview:

Students will investigate the parameters of the inverse square relationship, $y = Ax^B$, between light intensity and distance. A CBL and light intensity probe will be used to collect light intensity data for a light bulb of specific wattage at varying distances; the graphical calculator will be used to develop an appropriate mathematical model. Students will compare the parameters of the algebraic models for different wattages. They will predict the wattage of an unknown bulb from its intensity data. Students will work in cooperative learning groups to collect and analyze the data. Each group member will be assigned a specific task.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems by formulating a mathematical model for a real-world problem.
- **Communication** Students will demonstrate the ability to describe physical phenomena and mathematical ideas orally and in writing.
- **Reasoning** Students will make and test conjectures to interpret data.
- **Connections** Students will make connections among mathematical representations and between mathematics and science.
- **Algebra** Students will represent data graphically, numerically, and algebraically.
- **Functions** Students will understand the general properties and behavior of classes of functions, and model real-world phenomena with appropriate functions.
- **Statistics** Students will use curve fitting to make predictions from data.

Grade/Level:

Grades 10-12 (Algebra II, Pre-Calculus)

Duration/Length:

This lesson will take 3 periods (45 min.).

Prerequisite Knowledge:

Students should be able to do the following:

- Enter data into lists of the TI-82
- Display data stored in lists using STAT-plots
- Compare similar appearing classes of functions
- Determine the appropriate regression curve using the TI-82

Objectives:

Students will be able to:

- work cooperatively in groups.
- collect and organize data from resources.
- compare inverse power functions with similar appearing classes of functions.
- determine the exponent of a power function.
- determine the constant of variation for an inverse square regression curve.
- display data stored in lists using STAT-plots.
- compare inverse power functions with similar appearing classes of functions.
- determine the appropriate regression curve using the TI-82.

Materials/Resources/Printed Materials:

- CBL
- Light Intensity Probe
- Lamp
- Light bulbs of different wattages from 25 watts to 200 watts--1 per group
- Styrofoam block painted black
- Clothes pin
- 3 meter sticks
- Darkened hall or room
- Instructor's Notes
- Transparency of Class Data Sheet

per group:

- TI-82 graphics calculator (minimum of 1)
- Group Organization sheet (1)
- Student Worksheets 1 - 3 (5)
- Student Procedures Worksheet (1)

Development/Procedures:

- The teacher will explain the procedure for the groups and for the experiment, review the use of the CBL and lists on the TI-82, and organize the class into groups of three or four, assigning roles of group manager, calculator operator, CBL operator, and recorder.

- Distribute Worksheets 1 and 2 to each group member. Worksheet 1 will be completed by the groups while waiting for a turn with the CBL. Worksheet 2 will be used by the group taking its turn with the actual experiment.

- On Days 1 and 2, students will complete Worksheet 1, a comparison of the properties of the family of exponential functions with base less than one with the family of power functions with negative exponents, and a brief explanation of light intensity. One by one, groups will take a light bulb of a wattage not yet tested and perform the experiment described on Worksheet 2.

- Students will complete all parts of Worksheet 1 for homework after Day 1. Complete all parts by end of Day 1 and discuss with group on Day 2.

- By the end of Day 2, each group will have completed Worksheet 2 and produced a transparency of the best fit curve for its light bulb.

- On Day 3, distribute Worksheet 3 to each group member. The transparencies for all groups will be compared, and the data for each group's parameter will be presented. Each group will receive light intensity versus distance data for an unknown light-bulb. The groups will then complete Worksheet 3.

Evaluation:

Student achievement will be based on the student report sheets for each group.

Extension/Follow Up:

1. Explore the differences in light intensity for clear, frosted, and colored bulbs of the same wattage.

2. Explore the claimed intensity of "Miser" type light bulbs.

3. Require the students to fit both power and exponential functions to the data and justify the choice of a power fit with residual plots.

4. Explore the cause for the varying intensities observed at a single distance for a given light bulb. Include a physical explanation of the periodic, although not trigonometric, phenomenon. This occurs since the filament of the light bulb is heated by one cycle of electricity and then cools slightly before being heated by the next cycle.

5. Use three temperature probes to find the temperatures at different distances from the bulb, and fit a mathematical model relating temperature with distance from the light bulb. Since both light intensity and temperature are now related by the parameter of distance, temperature and light intensity could be related by eliminating the parameter.

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Group Name______________________________

Group Manager______________________________

Calculator Operator______________________________

CBL Operator______________________________

Data Recorder______________________________

WATTS UP, DOC?

GROUP ORGANIZATION SHEET

DUTIES OF GROUP MANAGER

Keeps the group focused on the task. Assists the calculator operator in interpretation of the calculator results. Assists the CBL operator in interpretation of CBL data. Assists the recorder in final wording of the report.

DUTIES OF CALCULATOR OPERATOR

Inputs information into the calculator. Helps interpret output of calculator. Assists the CBL operator in positioning the light intensity probe.

DUTIES OF CBL OPERATOR

Positions the light intensity probe on the meter stick. Takes readings from the CBL. Selects the appropriate reading at each distance.

DUTIES OF DATA RECORDER

Records data as it is generated. Completes and submits the group copy of the student worksheets in final form.

Name_______________________________

Group Name_______________________________

WATT'S UP, DOC?
WORKSHEET 1--PRE-LABORATORY EXPLORATION

I. POWER FUNCTIONS

Functions of the form $\mathbf{y = A\ x^B}$ are called power functions.

A. Use the graphing calculator to investigate changes in the graphs of the functions produced by letting A = 1 and graphing the function when B = 1, B = 2, B = 3, B = 4. When graphing, choose x values which are non-negative. Describe the shape of the graph in terms of x intercept, y intercept, and whether the graph is increasing or decreasing.

B. Use the graphing calculator to investigate changes in the graphs of the functions produced by letting A = 1 and graphing the function when B = -1, B =- 2, B = -3, B = -4. When graphing, choose x values which are non-negative. Describe the shape of the graph in terms of x intercept, y intercept, and whether the graph is increasing or decreasing.

C. Compare the graphs with positive and negative B values. Discuss the x-intercepts, y-intercepts, and whether the graph is increasing or decreasing. Which graph has both horizontal and vertical asymptotes? Which graph has an x-intercept and a y-intercept? If you knew that a graph was a power function, how could you tell the sign of B?

II. EXPONENTIAL FUNCTIONS

Functions of the form $\mathbf{y = A\ B^x}$ are called exponential functions.

A. Use the graphing calculator to investigate changes in the graphs of the functions produced by letting A = 1 and graphing the function when B = 2, B = 3, B = 4, B = 5. When graphing, choose x values which are non-negative. Describe the shape of the graph in terms of x intercept, y intercept, and whether the graph is increasing or decreasing.

B. Use the graphing calculator to investigate changes in the graphs of the functions produced by letting A = 1 and graphing the function when B = .2, B = .4, B = .6, B = .8. When graphing, choose x values which are non-negative. Describe the shape of the graph in terms of x intercept, y intercept, and whether the graph is increasing or decreasing.

C. Compare the graphs with B values greater than one with B values between 0 and 1. Discuss the x-intercepts, y-intercepts, and whether the graph is increasing or decreasing. When does the graph have a horizontal asymptote? Which graph has x an y intercepts? If you knew that a graph was an exponential function, how could you tell if B were greater than 1 or between 0 and 1?

III. COMPARE POWER FUNCTIONS AND EXPONENTIAL FUNCTIONS

A. If you had a graph of an unknown function, and you knew that it was either a power function with a positive exponent or an exponential function with a base greater than 1, how could you decide the type of the function?

B. If you had a graph of an unknown function, and you knew that it was either a power function with a negative exponent or an exponential function with a base less than 1, how could you decide the type of the function?

IV. Imagine that you had 100 M&M's distributed uniformly an a square sheet of paper.

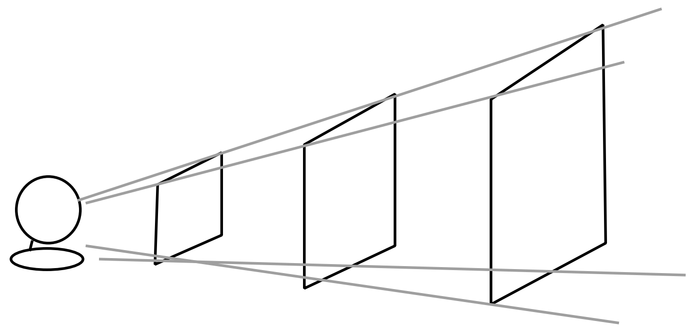
A. Complete the following chart relating the side of the square, the area of the square, and average number of M&M's per square meter.

Side in meters	Area in meters	number of M&M's / sq. meter
10		
8		
6		
4	(Add a chart of this format)	
2		
1		
0.8		
0.6		
0.4		
0.2		
0		

B. Use the calculator to plot the number of M&M's per square meters (y list) against the length of the side of the square in meters (x list). Sketch the graph below.

C. Explain why a power function is an appropriate fit for these data. Find the power function for this data, and explain the significance of the constants A (a on the calculator) and B (b on the calculator).

V. Light is emitted as discrete bundles of energy called photons. Photons can be thought of as discrete particles. An ordinary light bulb emits a fixed number of photons per second. Suppose that a particular light bulb is emitting photons so that 1000 photons are hitting an area of 1 square meter at a distance of 1 meter from the bulb. When the same 1000 photons travel to a distance of 2 meters from the bulb, they now cover an area of $2^2 = 4$ square meters. When the same 1000 photons are a distance of 3 meters from the bulb, they will cover an area of $3^2 = 9$ square meters.



A. Describe the area covered by the 1000 photons as they reach a distance D from the light bulb.

B. Intensity is a measure of the number of photons per unit of area. When the 1000 photons reach a distance 1 meter from the bulb, there are 1000 photons per square meter. When the 1000 photons reach a distance of 2 meters from the bulb, there are 1000/4 = 250 photons per square meter. When the 1000 photons reach a distance of 3 meters from the bulb, there are 1000/9 = 111.1 photons per square meter. How many photons per square meter will there be when the photons are a distance of 4 meters from the bulb? A distance of 10 meters from the bulb? A distance of D meters from the bulb?

C. Explain the relationship between the M&M problem and the light bulb problem.

D. What type of function would you expect to find as a model for intensity versus distance from the light bulb? Explain your choice.

Group Name________________________________

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WORKSHEET 2--DATA ANALYSIS

1. Read the Student Procedures Worksheet.

2. Conduct the experiment and record your data.

3. Record your data in your calculator, entering $\boxed{\text{STAT}}$ $\boxed{\text{EDIT}}$. Enter the cm distance into L_1 and the mW/cm^2 readings into L_2. Adjust your window to Xmin = 0, Xmax = 240, Xscl = 10; Ymin = 0, Ymax = 1, Yscl = .1 .

4. To display a scatter plot, enter $\boxed{\text{STAT PLOT}}$, being sure a stat plot is on, scatter plot is highlighted, and your data is represented in the correct lists.

5. Display a graph of your data. What type of function is represented by your graph? Explain your choice.

6. Determine the equation for your data by pressing $\boxed{\text{STAT}}$ > $\boxed{\text{CALC}}$ and selecting your regression formula. What is your correlation coefficient? Do you think you have selected the correct type of function? If not, select another regression and try again. Justify your decision.

7. Enter your function into $\boxed{Y =}$. With the cursor on $Y_1 =$, press $\boxed{\text{VARS}}$ $\boxed{5}$ > > to $\boxed{\text{EQ}}$ $\boxed{7}$. Your function is now pasted into Y_1. Does your curve fit the data? Print and make a transparency of your graph your graph.

8. What does the graphed data suggest to you when you physically set up an ideal study area in your home or in your room?

STUDENT PROCEDURES FOR DATA COLLECTION

Notes: 1. The light probe is one of the autoIDENT probes which does not require a program.
2. Avoid standing near the light probe if you are wearing bright/light-colored clothing.

1. Insert the light probe into channel 1 of the CBL unit. Turn on the CBL and the light source. You will automatically get a reading. You will be in the correct mode if there are no units of measure given behind the number on the display. If there is a unit of measure, press the mode button until the units disappear. Turn off all other room lights.

2. Point the light probe directly at the bulb. Adjust the position of the Styrofoam block to the nearest multiple of 10 cm so the CBL reads approximately .9. This reading is actually measured in mW/cm^2. (If your reading is .92 mW/cm^2 at 82 cm, start taking readings at 80 cm.)

3. Record both the distance and the reading on the Data Collection worksheet. The numbers displayed on the CBL will fluctuate. Why? Record the highest reading displayed for each position.

4. Move the block 10 cm further from the light source. Again record the highest reading.

5. Continue this procedure, taking additional readings in 10 cm intervals until your reading is less than .05 mW/cm^2 or you are further than 3 meters from the bulb.

6. Return to your seats so the next group may use the CBL.

STUDENT LABORATORY WORKSHEET GROUP________________

Distance from Light Source (cm)	High Reading (mw/cm^2)	Low Reading (optional)

CLASS DATA

DISTANCES in cm	WATTAGES							
	25	40	55	60	75	100	125	150
A =								
B =								
r =								

Group Name______________________________

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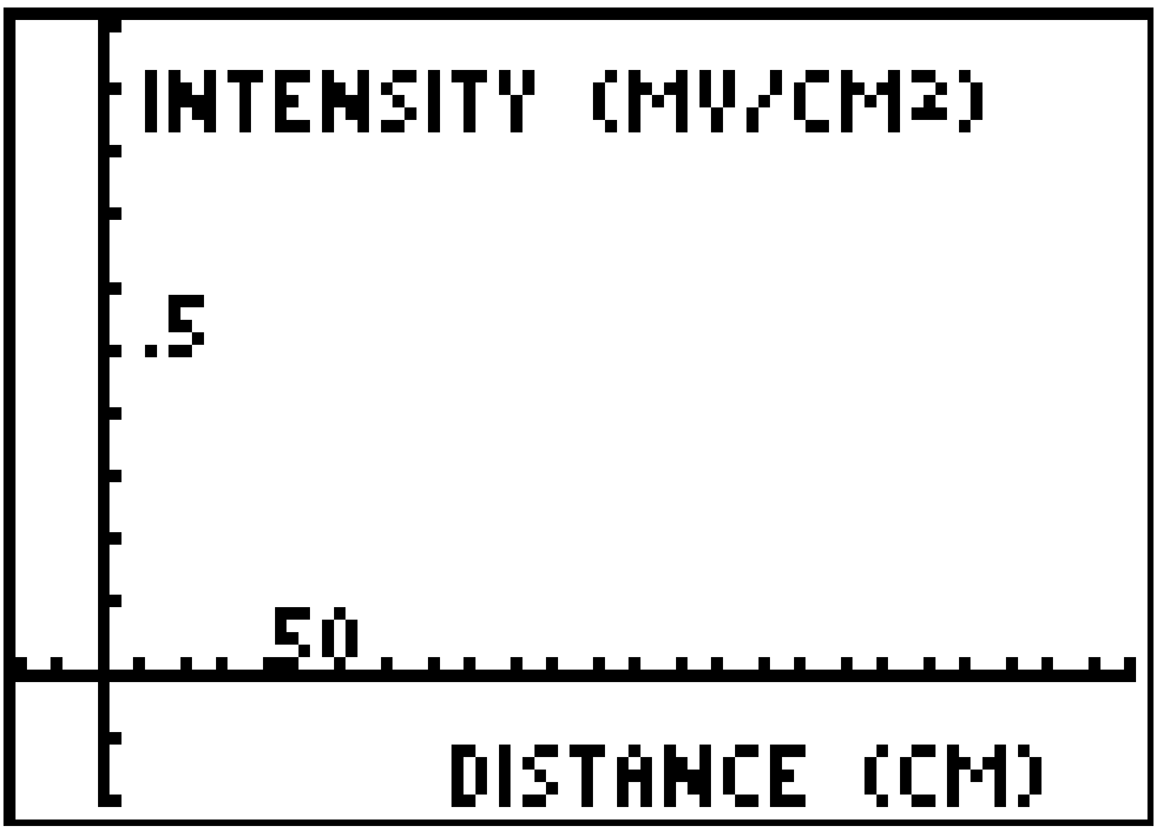
WORKSHEET 3--INVESTIGATION OF CLASS RESULTS

1. Sketch your best-fit curve on the graph below and those of at least two other groups. Label the wattage on each curve.



2. What do you notice about the graphs? Discuss their properties with your group and record your conclusions. Include in your discussion how the curves are the same and how they differ.

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3. Copy the class data for the variation **$Y = AX^B$** into the chart.

Chart

Group #	Bulb Wattage	A(mW)	B

4. Graph **A** against the bulb wattage.

5. Discuss in your group what type of function best fits this data and record your conclusions. Fit the curve your group chose to the data on the graph.

Algebraic model ____________________

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6. Decide the wattage bulb that would generate the lists below.

Bulb Wattage __________________

Data List

Distance	Intensity
50	.918
60	.732
80	.474
100	.321
120	.233
140	.178
160	.142
180	.115
200	.098

7. Discuss how your group arrived at this answer.

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Instructor's Notes

Laboratory Setup

1. Setup your experiment in a room which can be darkened.

2. Place three meter sticks directly in front of a light source. (See figure 1)

3. Clamp the light probe with a spring clothespin. Anchor it into a Styrofoam block which has been painted black and place the block on the first meter stick. The probe should be pointed directly at the bulb and at the same height.

4. Attach the light probe to channel 1 in the CBL.

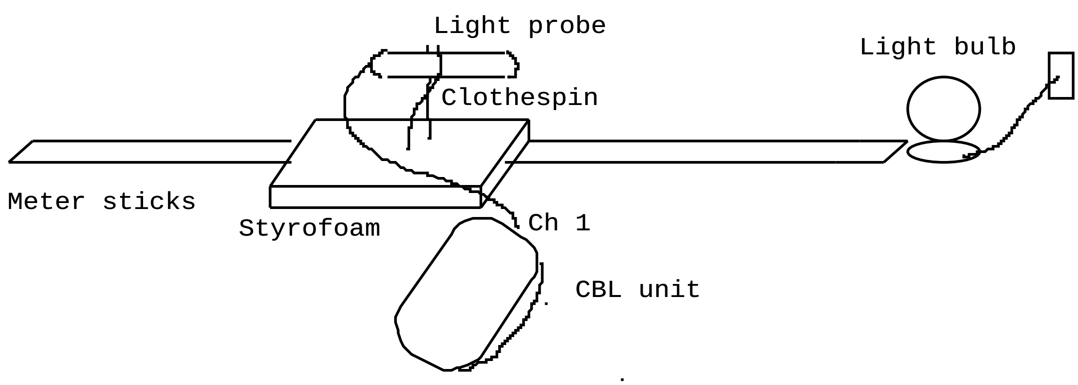

Figure 1

Instructions

1. The light probe is one of the autoIDENT probes, therefore a program is not required and measurements can be made directly from the CBL. The units for light intensity are milliwatts per centimeter squared.

2. Caution students that individuals with light clothing should avoid standing near the light bulb as this may cause the light to be reflected.

3. The range of the probe is 10 $\mu W/cm^2$ to $1 mW/cm^2$ so readings over .95 tend not to be accurate.

4. The intensity readings fluctuate periodically do to the nature of a filament bulb so the CBL needs to be observed over a short period of time and the highest reading recorded at each distance.

5. The experiment is written using only 1 CBL. If available, you may wish to set up more than one station to speed up the data collection process.